Software-based Erasure Codes for Scalable Distributed Storage

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Outline

• General motivation for distributed storage systems

• Approaches for protecting data using codes

• Implementation issues for graph-based codes
  – Encoding/Decoding with graph-based codes
  – Luby Transform (LT) Code
  – Lincoln Erasure Code (LEC)

• Experiments
  – Throughput comparison
  – Reliability comparison

• Summary
The Sensor Data Explosion

Data growth > 120,000X
Critical Technology Trends

• Follow the commercial technology trends in
  – Networking
  – Data Storage
  – Processing

•Capabilities in 2002
  – 10 Gbps Networks
  – 160 Gigabyte Disks

•Estimates in ~ 2010
  – 10 Tbps Networks
  – 40 Terabyte Disks

Credit: Scientific American
Distributed Storage System Architecture
Hardware for Distributed Storage

- **Commercial-off-the-shelf (COTS) Components**
  - Leverage economies of scale
  - Interoperable open standards
  - In the last year …
    Storage density has doubled …
    … and cost per GB is down 50%

- **Custom System Integration**
  - Aggregate low-cost storage nodes
  - Matched to unique user requirements
  - Performance can be scaled over time

![Trends in the Cost of Storage](chart.png)
Erasure Coding for Data Storage

- **Replication** *(N,1)*
  - No encoding, decoding required
  - Negative impact on available storage capacity

- **Single Parity Check** *(N+1,N)*
  - Simple encoding and decoding
  - Protects only against a single failure

- **Reed-Solomon** *(N,K)*
  - Minimum distance, but computationally intensive
  - Commercial hardware limited to small blocks
  - Software implementations are relatively slow

- **Graph-based Low-density Parity Check** *(N,K)*
  - Minimum distance traded for high performance
  - Software implementations are sufficiently fast
  - Scales to large block sizes (up to thousands)
Graph-based, Large Block Erasure Coding

- **Codes on Graphs**
  - Large blocks, low density
  - Bipartite: data, parity nodes
  - Heavy-tail degree distribution
  - Random assignment of edges

- **Simple serial/parallel encoder**
  - Steady throughput of 100’s Mbps

- **Iterative decoder**
  - Throughput depends on erasures
  - 100’s Mbps with maximum losses

- **Probabilistic erasure correction**
  - Tune to meet user requirements
  - ~5% overhead for $10^{-9}$ reliability

![Graph](image_url)
Encoding Data for Distributed Storage

1. IP Header
2. DSS Header
3. Data and Parity
4. Computing Parity:
   \[ a = 5E \oplus 93 \oplus 15 \]
An Iterative Decoder for Erasure Codes

- Draw graph, including erased nodes and adjacent nodes
- For each parity node of degree one, correct erased node

1. Original graph
2. Erased in Red
3. Remove erased parity and known data elements
4. Solve data using degree-one parity
5. Continue …
6. Last one!

DONE
Choosing a Graph Code

- **Specify the Block Size**
  - 7500 Elements

- **Define Data/Parity Split**
  - 5000 Data
  - 2500 Parity

- **Parity Degree Distribution**

<table>
<thead>
<tr>
<th>Degree</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1100</td>
</tr>
<tr>
<td>3</td>
<td>600</td>
</tr>
<tr>
<td>4</td>
<td>300</td>
</tr>
<tr>
<td>5</td>
<td>200</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>3000</td>
<td>1</td>
</tr>
</tbody>
</table>

- **Edge Assignment**
  - For parity of degree $d$, randomly select subset of $d$ data elements
Luby Transform (LT) Code

• **Similarity with our approach**
  – High performance graph-based coding approach

• **Differences**
  – Different motivation: LT sought reliable broadcast capability
  – Bits sent until file can be reconstructed
  – Sent no data bits
    - Good for broadcast application
    - Slower for storage application
Lincoln Erasure Code (LEC)

The number of parity nodes of degree \( j \) is given by the following equation:

\[
N_j(s, p, k, A, j^*) = s \sum_{j = j^* + 2}^{imax} \frac{Lk}{j - j^* - 1} \frac{Lk + A \sqrt{Lk}}{j}
\]

for \( j = j^* + 2, j^* + 3, \ldots, imax \)

where \( s \) is a scaling factor, \( p \) is the fraction of corruption being protected against, \( k \) is the number of data nodes, \( A \) is a variable, and \( L \) is the desired loss protection.
Experimental Plan

- Throughput comparison
- Reliability comparison
  - File size: 13.1 Mbyte
  - Loss rates
    - 1%
    - 10 %
    - 20 %
    - 50 %
    - 75 %
  - Block size
    - 5040 data
    - 2520 data
  - Targeted probability of unsuccessful file reconstruction
    - $10^{-6}$
    - $10^{-4}$
## Throughput Comparison of Erasure Codes

<table>
<thead>
<tr>
<th>Erasure Coding Method</th>
<th>Encoding (Mbps)</th>
<th>Decoding (20% Loss) (Mbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEC</td>
<td>240</td>
<td>760</td>
</tr>
<tr>
<td>Luby Code</td>
<td>180</td>
<td>220</td>
</tr>
<tr>
<td>Reed-Solomon</td>
<td>0.09</td>
<td>0.20</td>
</tr>
</tbody>
</table>
Reliability Comparison of Erasure Codes

Block of 5040 data and 1760 parity elements
Comparison of Reliability with Various Percent Loss and 5040 Data

### 5040 data, targeting $10^{-6}$ reliability

<table>
<thead>
<tr>
<th>Percent Loss</th>
<th># Parity Elements</th>
<th>LEC Reliability</th>
<th>Luby Reliability</th>
<th>Minimum # Parity Elements Needed</th>
<th>Overhead</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>130</td>
<td>$6.5 \times 10^{-7}$</td>
<td>1.0</td>
<td>51</td>
<td>2.55</td>
</tr>
<tr>
<td>10</td>
<td>860</td>
<td>$3.3 \times 10^{-7}$</td>
<td>$6.1 \times 10^{-1}$</td>
<td>560</td>
<td>1.54</td>
</tr>
<tr>
<td>20</td>
<td>1760</td>
<td>$8.3 \times 10^{-7}$</td>
<td>$6.1 \times 10^{-3}$</td>
<td>1260</td>
<td>1.40</td>
</tr>
<tr>
<td>50</td>
<td>6350</td>
<td>$1.7 \times 10^{-6}$</td>
<td>$3.5 \times 10^{-6}$</td>
<td>5040</td>
<td>1.26</td>
</tr>
<tr>
<td>75</td>
<td>20000</td>
<td>$2.2 \times 10^{-6}$</td>
<td>$2.8 \times 10^{-7}$</td>
<td>15120</td>
<td>1.42</td>
</tr>
</tbody>
</table>

### 5040 data, targeting $10^{-4}$ reliability

<table>
<thead>
<tr>
<th>Percent Loss</th>
<th># Parity Elements</th>
<th>LEC Reliability</th>
<th>Luby Reliability</th>
<th>Minimum # Parity Elements Needed</th>
<th>Overhead</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>110</td>
<td>$1.4 \times 10^{-4}$</td>
<td>1.0</td>
<td>51</td>
<td>2.16</td>
</tr>
<tr>
<td>10</td>
<td>810</td>
<td>$2.3 \times 10^{-5}$</td>
<td>1.0</td>
<td>560</td>
<td>1.45</td>
</tr>
<tr>
<td>20</td>
<td>1700</td>
<td>$3.5 \times 10^{-5}$</td>
<td>$2.9 \times 10^{-2}$</td>
<td>1260</td>
<td>1.35</td>
</tr>
<tr>
<td>50</td>
<td>6230</td>
<td>$9.9 \times 10^{-5}$</td>
<td>$1.7 \times 10^{-5}$</td>
<td>5040</td>
<td>1.24</td>
</tr>
<tr>
<td>75</td>
<td>19250</td>
<td>$1.1 \times 10^{-4}$</td>
<td>$7.5 \times 10^{-7}$</td>
<td>15120</td>
<td>1.27</td>
</tr>
</tbody>
</table>
### Comparison of Reliability with Various Percent Loss and 2520 Data

#### 2520 data, targeting $10^{-6}$ reliability

<table>
<thead>
<tr>
<th>Percent Loss</th>
<th># Parity Elements</th>
<th>LEC Reliability</th>
<th>Luby Reliability</th>
<th>Minimum # Parity Elements Needed</th>
<th>Overhead</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80</td>
<td>$1.9 \times 10^{-6}$</td>
<td>1.0</td>
<td>25</td>
<td>3.20</td>
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<tr>
<td>10</td>
<td>470</td>
<td>$2.3 \times 10^{-6}$</td>
<td>$9.0 \times 10^{-1}$</td>
<td>280</td>
<td>1.68</td>
</tr>
<tr>
<td>20</td>
<td>950</td>
<td>$1.0 \times 10^{-6}$</td>
<td>$3.2 \times 10^{-2}$</td>
<td>630</td>
<td>1.51</td>
</tr>
<tr>
<td>50</td>
<td>3360</td>
<td>$7.3 \times 10^{-7}$</td>
<td>$6.8 \times 10^{-6}$</td>
<td>2520</td>
<td>1.33</td>
</tr>
<tr>
<td>75</td>
<td>10400</td>
<td>$1.8 \times 10^{-6}$</td>
<td>$1.1 \times 10^{-6}$</td>
<td>7560</td>
<td>1.37</td>
</tr>
</tbody>
</table>

#### 2520 data, targeting $10^{-4}$ reliability

<table>
<thead>
<tr>
<th>Percent Loss</th>
<th># Parity Elements</th>
<th>LEC Reliability</th>
<th>Luby Reliability</th>
<th>Minimum # Parity Elements Needed</th>
<th>Overhead</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>65</td>
<td>$1.4 \times 10^{-4}$</td>
<td>1.0</td>
<td>25</td>
<td>2.60</td>
</tr>
<tr>
<td>10</td>
<td>435</td>
<td>$1.4 \times 10^{-4}$</td>
<td>1.0</td>
<td>280</td>
<td>1.55</td>
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<tr>
<td>20</td>
<td>900</td>
<td>$5.8 \times 10^{-5}$</td>
<td>$2.9 \times 10^{-1}$</td>
<td>630</td>
<td>1.43</td>
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<tr>
<td>50</td>
<td>3240</td>
<td>$3.6 \times 10^{-5}$</td>
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<tr>
<td>75</td>
<td>10100</td>
<td>$2.0 \times 10^{-4}$</td>
<td>$1.8 \times 10^{-6}$</td>
<td>7560</td>
<td>1.34</td>
</tr>
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Summary

• Motivated need for codes in storage systems

• Introduced Lincoln Erasure Codes (LEC)
  – Defined
  – Described implementation of code in storage systems
  – Demonstrated throughput and reliability performance